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Experimental Survey Roterdam Demand System By Using Urban Family Of Consumer Costs Data Case Study: Province Of Sistan and Balouchestan, Iran

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ABSTRACT: Consumer demand system expresses how consumers allocate their income among various products. These models are usually based on microeconomic theory which considers the demand side and neglects the supply side. In other words, the demand-supply analysis is independent. In this research, firstly, the characteristics of consumer demand and the demand function extraction methods are discussed and finally, using data from the spending of urban households in Sistan and Blouchstan both bound and unbound Rotterdam demand system is estimated. The results show that the 2 states of theory of the demand is true for goods groups and to determine the Rotterdam demand system consistent with the theory or not, homogeneity and symmetry constraints of the parent test, are tried. The results indicated that the homogeneity and symmetry in Rotterdam demand system is provided.

***Keywords*:** Roterdam Demand System, Urban Household Expenditure, Consumer Costs, Econometrics Model.

INTRODUCTION

The issue of allocation, that is almost started simultaneously with microeconomics, is the specific optimal allocation of money among different options and in its dual sense it means to minimize the amount of money used to achieve a givens set of goals (e.g. a certain level of utility). Allocation models have been formulated not only for consumer demands, but for several cases such as the demand for factors of production, allocation of imports demand, distribution of portfolio investment and distribution of the land area among various products. In all these models, the main argument is how objective function is achieved to an optimal point based on a series of relevant variables which may be useful or not useful. This optimization of various systems specifies the curvature of objective function or consequential form of objective function and related provisions. Although these systems are based on the economic theory of individual behavior, it is often applied for the behavior of entire market of the whole economy. In fact, the aggregation of individual behavior for explaining the entire behavior can be made by imposing a set of specific assumptions.

This article aims to examine the characteristics of demand model (ROTERDAM, AIDS, CBS, NB) as well as experimental estimation of Rotterdam demand systems (constrained and unconstrained) based on the data of the expenditures of urban household of Sistan and Balouchestan province over the period 1985-2009. In this respect, the theoretical basis of consumer demand will be firstly indicated and then the characteristics of the regarded application systems will be explained and statistically analyzed in continuous. In this analysis, homogeneity constraint test and demand systems symmetry will be discussed.

Theoretical characteristics of consumer demand

In this section, the basic features of theory of consumer demand are reviewed. According to microeconomic literature, consumer preferences can be wrote under appropriate assumptions as a utility function:
u= u(qi…,qn)

In that q represents the amount of goods and n is the number of commodities. This function is increasing based on q_i values which are strictly quasi-concave and it is generally assumed that it has primary and secondary derivatives. Vector of first order derivatives $(u_q = [\partial u / \partial q_i])$ includes n vector of final desirability that is positive due to the increasing nature of u(q) based on q=[q_i]. n×n matrix of second order derivatives of $(\coprod = \lfloor \partial^2 u / \partial \infty d \rfloor)$ is a symmetrical derivate. Given the strictly quasi-concave property of equation (1), we have:

$$x' \coprod x \le 0 \quad , \quad \forall x \ne 0 \quad , u'_q . x = 0 \qquad (2)$$

The amount of consumed money (m) is non-zero, but is limited that is used for applying $p_i q_i$ i=1,…n& to create the desirability resulted from the products. P_i is the unit price of i[th] goods. This allocation of money is completely conducted between the expenditures of various goods ($P_i q_i$); thst is to say, the following equation should b as follows:

$$\sum_i p_i q_i = m$$

The issue of consumer optimization is that q vector is chosen among alternative vectors in such a way that the utility level of equation (1) be maximized with respect to the constraint (3). The mathematical solution of the above optimization will give us the following first order conditions:

$$u_q = \lambda p$$

In that λ (as Lagrange coefficient) is positive p=[p_i] is a n-rows vector of prices. Considering the set of equations (3) and (4), optimal value of q can be obtained. Marshal demand function I achieved by solving this matrix that can be generally written as follows:

qi= fi(m,p1,…,pn) & i=1,…,n (5)

If there wants to be an intermediate solution for quantities of commodities, the strictly quasi-concave condition of equation (2) should be established as $x' \coprod x \le 0 , \forall x \ne 0 , u'_q . x = 0$ that is known as strong quasi-concave.

Sometimes the differentiation of Marshall demand functions (5) is of interest to economists, because many models that have been estimated and evaluated in practice are differentiable. Meanwhile, many features and implications of theory of consumer demand may be better displayed as demand elasticities that require being differentiated of demand function. For example, the logarithmic and differential form of equation (5) can be written as follows:

$$d1n \ \ q_i d1n \ m + \sum_j \mu_{ij} d1n \ p_j \qquad i = 1,..., n \qquad (6)$$

Where n_i is the income elasticity of demand for i[th] goods and μ_{ij} is the elasticity of i[th] than the price of j[th] goods. These elasticities should have a series of specific features to be consistent with the demand theories.

If budget share is $w_i = \frac{p_i q_i}{m}$, it should be $\sum_i w_i = 1$ with respect to equation (3). Some of these features can be defined as follows based on Aslatsky elasticity or compensatory price elasticity, ε_{ij}:

$$\varepsilon_{ij} = \mu_{ij} + \eta_i w_j \qquad (7)$$

Assuming that the utility level of consumer is stable, these elasticities measure the consumer sensitivity to price changes. For the demands that are obtained based on budget equation (3), additive requirement ensures that:

$$\sum_i w_i \eta_i = 1 \qquad \text{Total parasite (1-8)}$$

$$\sum_i w_j \mu_{ij} = -w_j \qquad \text{Total curnutt (2-8)}$$

Which the following equation will be made by adding up the two equations and using equation (7):

$$\sum_i w_i \varepsilon_{ij} = 0 \qquad \text{Total Aslatsky (3-8)}$$

The other feature is homogeneity condition that equation (3) should be linear based on m,p_i

$$\sum_j \mu_{ij} = -n_i$$

$$\sum_j \varepsilon_{ij} = 0$$

Another significant experimental feature is Aslatsky symmetry:

$w_i \varepsilon_{ij} = w_j \ \varepsilon_{ji}$

The next feature is negative condition:

$\sum_i \sum_j x_i \varepsilon_{ij} x_j < 0$ $\quad x_i, x_j \neq$ Constant value

Aslatsky elasticities may show a certain structure of order of preferences or unity function. If the order of preferences is shown by utility function in that it is as the sum of n function that each one is only a function of goods, then:

$\varepsilon_{ij} = \varphi n_i (\partial_{ij} - \eta_j w_j)$ (total authority)

In that φ is the reverse of something that is called "monetary flexibility" and ∂_{ij} is Kronecker delta. If all the goods in the consumer's consumption basket is classified according to non-interference groups and utility function be a function of separable utility function for both groups, then if i goods is a part of f group and j goods belongs to G group and $F \neq G$, we will then have:

$\varepsilon_{ij} = -\varphi_{FG} \eta_i \eta_j w_j$ (weak separable):

That is true for all junctions between commodity groups of G and F $\varphi_{FG} = \varphi_{GF}$, feature (13) shows the weak separation. For strong separation among groups, $\varphi \ \varphi_{FG=}$ should be true, that is, it should be the same for all group junctions. It is clear that if all groups consist of only and only one goods, then one of the states of complete authority will be established:

Separation is useful in practical works, because it allows the special system of each group are individually formulated by assuming the specification of the money that will be spent for this group:

Allocation of money among commodity groups in a model with a higher level of allocation is determined only by the characteristics of the groups. Homogeneity preferences indicate this feature:

$$\eta_j = 1, \qquad \forall i \tag{14}$$

which indicates that the budget portion of i^th commodity does not change with changing income. Another concept that its usefulness has been proved is indirect utility function:

$$u^* = u(m, p_1, ..., p_n)$$

That is obtained by replacing q_i from equation (5) in equation (1) and its differentiation form can be written as follows:

$$du^* = \sum i(\partial u / \partial q_i)\left[(\partial f_i / \partial m)dm + \sum(\partial f_i / \partial p_i)dp_j\right]$$

$$= \lambda m\left(\sum i\, w_i \eta_i d1nm + \sum_i \sum_j w_i \mu_{ij} d1n\, p_j\right)$$

$$= \lambda m\left(d1nm \sum_j d1n\, p_j\right)$$

Which first order condition (4) and additive function (1-8) and (2-8) have been used. This equation shows that Lagrange coefficient of equation (4), λ, is the same of the final budget utility, $\partial u^* / \partial m$. Demand function can be obtained using the above rule:

$$q_i = -(m / p_{ij})\left(\frac{\partial u^*}{\partial 1n\, p_i} \Big/ \frac{\partial u^*}{\partial 1nm}\right)$$

In equation (16), $d1nm + \sum_j w_j d1n\, p_j$ can be considered as the kind of real income changes. $\sum_j w_j d1n\, p_j$ is the change of price index that is used to mediate m. The constancy of real income means lack of utility changes. Another way to examine the concept of real income is that it starts from the logarithmic differential equal o to budget (3).

$$d1nm = \sum_j w_j d1n\, p_j + \sum_j w_j d1n q_j$$

Then, it can be written that:

$$\sum_j w_j d1n q_j = d1nm - \sum_j w_j d1n\, p_j$$

In that left side variable i.e. the change in quantity, corresponds to real income in right side and as a result the following signs have been used:

$d1nQ=\sum_j w_j \, d1nq_j$ & $d1n\ p=\sum_j w_j \, d1n\, p_j$

That shows quantity index and Divisia price index. By its replacement in equation (18), we will have:
d 1 n m= d 1n P+ d 1 n Q
another concept that has been used in practice is expenditure functions that which, then:
if m expresses in terms of p,u, then:
m= e(u, $p_1,\ldots, p_n$)
This equation shows the minimum expenditure required to achieve utility level (u) with specific prices of $p_1,\ldots,p_n$. Using equation (16), its differential form can be written as follows:

$$d1ne=[1/(\lambda m)][du+\sum_j w_j \, d1n\, p_j]$$

That is used as a basis for Shephard formula:

$$w_i=\frac{\partial 1ne}{\partial 1n\, p_j} \qquad i=1,\ldots,n$$

Which gives demand equation of Hiski kind and is as follows:
$Q_i= h_i(u, p_1,\ldots,p_n)$ i= 1,…,n
If u is replaced in equation (25) using $u(m,p_1, \ldots, p_n)$, Marshall demand equations will be achieved as follows:

$$d1nq_i=[1/(\lambda m)]\eta_i \, du+\sum_i \varepsilon_{ij} \, d1n\, p_j$$

That is the form of logarithmic differentiation of equation (25). Accordingly, the nature of ε_{ij} as constant price elasticities of utility can be clearly seen.

Techniques of demand functions extraction

In econometrics, the ideal specification is to be ideally consistent with economic theory, its estimation is be and is suitable to observed data to be predicted with less error. A reasonable balance should be established among these three characteristics in choosing the model. The demand model should be consistent with the features mentioned in the previous section in the formulation of consumer allocation system. Although these properties for individual consumers, it can be established for the average or total agents. In general, different methods of extraction can be classified in four methods to achieve the demand equations that supply characteristics examined in the previous section.

Method 1: it initiates the extraction of demand equations with the explanation of consequential form of utility function as an increasing and quasi-concave function. Then it obtains the maximizing function of utility function according to budget constraint (3). For this purpose, it solves q values as a function of price and income using the equations of first order condition which gives us the demand function. In this method, utility function parameters are consistent with obtained demand equations.

The best example of this method is a linear expenditure system (LES). In this system, the basic utility function can be written as:

$$u=\sum_i \beta_i \,\ell n(q_i-\gamma_i) \quad , \ \sum_j \beta_j=1 \ , \ \gamma_i<q_i$$

Equations resulting from the demand function are as follows:

$$q_i=\gamma_i+(\beta_i/p_i)(m-\sum_j p_j\gamma_j)$$

The additive feature of equation (27) clearly shows the assumption of full independence of preferences orders. This function is rather empirically limiting and it is not also easy to estimate, for γ_j has been appeared in non-linear as β_i in all equations in equation (28). Meanwhile, the estimated γ_j should be less than the least amount of q_i in order not to be easily observed by the data.
System of demand equations was first estimated by Eston in an ideal way and its optimal equation lasted until Parks and Solari. Generally, it is clear that a completely specified utility function cannot be resulted in an interesting demand function.
Method 2: it starts from an explanation of a consequential form of indirect utility function and uses Roy rule to achieve demand function that can be estimated. An obvious example in this method is Translog indirect utility function that has been proposed by Christiansen et al:

$$u^*=\alpha+\sum_i \beta_i \,\ell n(p_j/m)+1/2\sum_i\sum_j \beta_{ij}\ \ell n(p_i/m)\ell n(p_j/m)$$

Considering $\beta_{ij}=\beta_{ji}$ & $\sum_i \beta_i=-1$, it can be concluded that:

$$w_i = \frac{\beta_i + \sum_j \beta_{ij} \ \ell n(p_j / m)}{-1 + \sum_k \sum_j \beta_{kj} \ \ell n(p_k / m)}$$

This system is also based on non-linear parameters and it is also not easy to be estimated. Moreover, it is impossible to meet this requirement that u and m be uniformly increasing or be declining for all possible prices and m in terms of the general price level. This way is impossible for prediction and simulation works. Also, it is probable that the prediction value of quantities is negative. Income elasticity related to equation (30) is as follows:

$$\eta_i = 1 - (\sum_j \beta_{ij} / w_i - \sum_k \sum_j \beta_{kj}) / x$$

In that x is the denominator of (30). For Aslatsky elasticity, ε_{ij}, its multiplication in W_i will also be as follows:

$$w_i \varepsilon_{ij} = (\beta_{ij} - w_i \sum_k \beta_{jk} + w_i \ w_j \sum_k \sum_i \beta_{kl}) / x$$

Which total Aslatsky conditions (1-8) provides homogeneity conditions of (2-9) and symmetry condition (10). However, controlling the sign β_{ij} cannot guarantee the negative condition (10). On the other hand, if the separable conditions of (12) or (13) is compared with equation (31) of this demand function, it will be clear that creating a separable direct utility function from the obtained demand function (30) is a relatively complex issue. This system is made homogeneous by placing $\sum_j \beta_{ij} = 0$ for all i_{th}. This property can be imposed or tested to the system or both of them may be created without essential changes in specifying the function.

Method 3: it is based on the specification of expenditure function (22). Hicks demand equations can be obtained as a function of utility level (unobservable) using Shephards lemma that its equivalent can be inserted by p,m values instead of utility level and remove it from the demand functions. The best example of this kind of specification is Almost Ideal Demand System (AIDS). To extract demand equations in AIDS demand system, a consumer expenditures function e(u,p) can be used as PIGLOG. PIGLOG function is as follows:

$$1n \ e(u,p) = (1-u).1n\{a(p)\} + (u).1n\{b(p)\}$$

In this equation, it is assumed that u is between zero and one in that zero is living in the minimum livelihood and one indicates utmost joy of life. .a(p) indicates the livelihood expenditure and b(p) indicates is the welfare costs which is defined as follows:

$$\ln b(p) = a_0 + \sum_k a_k . \ln p_k + 1/2 \sum_k \sum_j \gamma_{kj} * . \ln p_k . \ln p_j \quad (34)$$

$$\ln b(p) = \ln a(p) + \beta_0 \prod_k p_k^{\beta_k} \quad (35)$$

Therefore, the equation of direct cost AIDS will be as follows:

$$\ln e(u,p) = a_0 + \sum_k a_k . \ln p_k + 1/2 \sum_k \sum_j \gamma_{kj} * . \ln p_k . \ln p_j + u \beta_0 . \prod_k p_k^{\beta_k} \quad (36)$$

Where $\gamma^*, \beta_i \, \delta \, a_i$ are the parameters and it can be easily examined that e(u,p) is linear in terms of homogeneous p. if we have:

$$\sum_i a_i = 1 \ , \sum_j \gamma_{kj}^* = \sum_k \gamma^*_{kj} = \sum_j \beta_j = 0$$

The demand of various goods can be deduced from e(u,p) function using Shephards lemma. According to Shephards lemma, the equation $\frac{\partial e(u,p)}{\partial p_i} = q_i$ is established in that if both sides are multiplied in $\frac{p_i}{e(u,p)}$, we will have:

$$\frac{\partial \ln e(u,p)}{\partial \ln p_i} = \frac{p_i q_i}{e(u,p)} = w_i \quad (37)$$

Which, w_i is the budget proportion of i^{th} goods. If equation (36) is differentiated logarithmically, w_j will be obtained din the right side.

$$w_i = a_i + \sum_j \gamma_{ij} \ln p_j + \beta_i . u . \beta_0 \prod p_k^{\beta_k} \quad (38)$$

In which

$$\gamma_{ij} = \frac{1}{2} (\gamma_{ij}^* + \gamma_{ji}^*) \quad (39)$$

The utility maximize for the entire m expenditures from the perspective of consumer is (a.p). This equality can give form u as a m,p function which is an indirect function. If this issue is conducted for function (36) and is replaced in (38), then the proportion of ith expenditure proportion can be obtained as a function of m and p.

$$w_i = a_i + \sum_j \gamma_{ij} \ln p_j + \beta_i \ln\{m/p\} \quad (40)$$

In which:

$$\ln p = a_0 + \sum_k a_k . \ln p_k + 1/2 \sum_j \sum_k \gamma_{kj} . \ln p_k . \ln p_j \quad (41)$$

This function is called AIDS demand in the form of budget proportion in that the following relations are established:

$$\sum_{i=1}^{n} a_i = 1 \ , \sum_i \gamma_{ij} = 0 \ , \sum_i \beta_i = 0 \quad (42)$$

$$\sum_j \gamma_{ij} = 0 \quad (43)$$

$$\gamma_{ij} = \gamma_{ji} \quad (44)$$

AIDS system is not easy to interpret. This system shows that the proportion of the regarded goods will remain unchanged if there is no change in relative prices and real income (real money). Changes in real expenditures affects the proportion of goods' expenditure by β_i and the change in relative prices affects by α_i . β_i is positive for deluxe goods and zero for all negative goods. Also, it can be shown that AIDS equations systems can be generalized for all community.

The important point in this system is that given the price index p, the above equation is based on non-linear coefficients and the system almost forms Non-linear Almost Ideal Demand System (NADIS) and non-linear methods are also used for evaluating the coefficients and this issue by itself requires sufficient information and statistics. Eston index has been used as an alternative for the real index P in most empirical studies and then the system is appeared as a Linear Almost Ideal Demand System (LAIDS) and the demand function becomes a linear function of prices and the overall expenditures that can be evaluated using linear methods. Deitoun and Moulbar introduced Eston index to change their demand system to a linear system as follows:

$$\log p = \sum_k w_k \log p_k \quad (45)$$

Income elasticities of $\eta^1 i$ and domestic price elasticities of $\mu^1 ij$ and the demand system of LAIDS is calculated as follows:

$$\eta_i^1 = \frac{\beta_i^1}{w_i} + 1 \quad (46)$$

$$\mu_{ii}^1 = \frac{\gamma_{ii}^1}{w_i} - 1 \quad (47)$$

$$\mu_{ii}^1 = \frac{\gamma_{ij}^1}{w_i} \quad (48)$$

It can be shown that the income elasticity of η_i and domestic price elasticities of $\mu^1 ij$ and the demand system of LAIDS are as follows:

$$\eta_i = \frac{\beta_i}{w_i} + 1 \quad (49)$$

$$\mu_{ii} = \frac{\gamma_{ii}}{w_i} - 1 - \beta_i \quad (50)$$

$$\mu_{ij} = \frac{\gamma_{ij}}{w_i} - \beta_i (w_j / w_i) \quad (51)$$

Method 4: many empirical studies related to demand have been recently conducted with respect to bilateral logarithm and elasticity. These studies show good results empirically; however, they are not suitable in terms of theoretical conditions that are mentioned in the previous section. As mentioned, apart from homogeneous conditions, these conditions can be indicated based on elasticities. Meeting the properties of function elasticities requires constant budget proportion that are not interesting theoretically and are acceptable empirically. Taile (1965) started from an explanation of bilateral logarithm as (6) in that μ_{ij} was inserted using equation (6) by ε_{ij}. By multiplying the two sides in w_i, we will have:

$$w_i.d\ln q_i = b_i(d\ln m - \sum_j w_i d\ln p_j) + \sum_j s_{ij} d\ln p_j \quad (52)$$

In that $s_{ij} = w_i.\varepsilon_{ij}$ & $b_i = w_i.\eta_i$ acts as a constant value. These constant selections are known as Rotterdam system. In a conducted review, total Engel- Aslatsky shows that:

$$\sum_i b_i = 1 \qquad \sum_i s_{ij} = 0 \quad (53)$$

While homogeneity conditions are obtained by the following equation:

$$\sum_j s_{ij} = 0 \quad (54)$$

and symmetry condition changes to the following equation:

$$s_{ij} = s_{ji} \quad (55)$$

The negative condition of semi-definite will be as follows:

$$\sum_i \sum_j x_i s_{ij} x_j < 0 \qquad x_i, x_j \neq constant \quad (56)$$

All these conditions are based on system constants and can be tested or imposed on the system. Another interesting feature of the selections out of these parameters is that specific preferences structure has specific states. To reach a complete independence, it should:

$$s_{ij} = \varphi.b_i(\delta_{ij} - b_j) \quad (57)$$

While for weak separation, it indicates as follows:

$$s_{ij} = \varphi_{FG}.b_i b_j \quad (58)$$

In that i and j belongs to F and G groups, respectively and for strong replacement, φ_{ij} is replaced by φ. Since $\eta_i = b_i / w_i$, homogeneity can be obtained only by imposing b_i / w_i for all I, i.e. by making w_i constant to all price changes.

This model is a public state for it is shown with this problem that mutual reactions of i and j are shown by s_{ij}. Regarding $b_i / w_i = \eta_i$, η_i sign is shown by b_i. An estimated goods may be postal ($\eta_i < 0$, $b_i < 0$) or non-postal ($b_i \geq 0$, $\eta_i \geq 0$). In the second state, the goods can be normal ($b_i \leq w_i$ & $\eta_i \leq 1$) or a deluxe one ($b_i > w_i$ & $\eta_i > 1$). The goods can be changed from deluxe to normal and vice versa by changing wi . One goods cannot be changed from a postal one to a non-postal one. It can be concluded from equation (46) that β_i sign determines whether η_i is bigger than the other one or not. A goods is either deluxe or required without the possibility that it is changed from an extrovert variable. Each constant value of bi is appeared from Rotterdam system and β_i is appeared from AIDS system to be useful. Is it possible that an specification is appeared in such a way that a goods passes from an economic life cycle i.e. first deluxe one, then the normal one and finally is posted. It can be rarely seen with a normal level of an accumulation of a postal goods, although the reduction of its practical significance is the limitation of b_i constant.

Of the four methods surveyed, the first one (that is specifically formulated from a direct utility function) has the minimum of attraction, for it is not resulted in an interesting demand system.

Differential demand functions family

Rotterdam model (52) can be written using equations (19) and (20) as follows:

$$w_i.d\ln q_i = b_i.d\ln Q + \sum_j s_{ij} \ln p_j \quad (59)$$

This is in fact one of the four models studied in the previous section. Now consider AIDS model of equation (40) that is a differential model. If $d\ln p^*$ of equation (40) as $d\ln p$ is replaced by equation (20), using equations (19) and (20), we will have:

$$dw_i = \beta_i.d\ln Q + \sum_j \gamma_{ij} \ln p_j \quad (60)$$

If equations (59) and (60) are considered, the right sides are very similar to each other and its left side is different; but they are related to each other. In fact, it can be written that:

$$dw_i = w_i.d\ln q_i + w_i.d\ln p_i - w_i.d\ln m \qquad (61)$$

It shows that $w_i.d\ln q_i$ is a part of budget proportion change of w_i, while $w_i.d\ln p_i$ and $-w_i.d\ln m$ relates to extrovert changes in price and money. It can be shown using model (16) that how coefficients (59) and (60) are related to reach other and the right side of equation (59) is obtained by inserting $w_i.d\ln q_i$ in equation (61):

$$dw_i = b_i.d\ln Q + \sum_j s_{ij} d\ln p_j + w_i d\ln p_i - w_i$$

$$= (b_i - w_i)d\ln Q + \sum_j (s_{ij} + w_i\delta_{ij} - w_i w_j)d\ln p_j \qquad (62)$$

Where equations (20) and (21) have been used to be inserted in dlnm. Its comparison with equation (60) shows that its equivalent is as follows:

$$\beta_i = b_i - w_i$$

$$\gamma_{ij} = s_{ij} + w_i\delta_{ij} - w_i w_j \qquad (63)$$

If w_i is considered as a variable and s_{ij},b_i as a constant, the main difference is to consider γ_{ij}, β_i as a constant. In fact, the two systems can be compared with each other as they are different.

CBS.NB$_R$ Demand Systems

Theoretical bases of this system were founded by "Drill" and "Kler" and were reviewed later by people like Barten, Zilenburgh, Nedal, Drill, Philip G and Dess Champs.

Drill and Kler, (1985) made a hybrid or combinational model of Almost Ideal Demand System of Deitoun and Moulbar and Rotterdam demand system (Theil, 1975) in Central Statistics Organization of Netherlands using the replacement of $\beta_i + w_i$ instead of b_i in equation (59) and substituting $w_i d\ln Q$ to the left side. The result of such system (CBS) can be written as follows:

$$w_i(d\ln q_i - \ln Q) = \beta_i d\ln Q + \sum_j s_{ij} d\ln p_j$$

$$w_i(d\ln q_i / Q) = \beta_i d\ln Q + \sum_j s_{ij} d\ln p_j \qquad (64)$$

In that v parameters are supposed constant and q_i is the demand value of i^{th} goods and P_j is the price of j_{th} goods. Q is the total real expenditure and is defined as follows:

$$d\log Q = \sum_{j=1}^{n} w_j d\log q_j = d\log m - \sum_{j=1}^{n} w_j d\log p_j \qquad (65)$$

In this equation, m is the value of total expenditure and $w_i = p_i q_i / m$ is the budget proportion of i_{th} goods and n is the number of goods. Price coefficients S_{ij} that is also called Aslatsky coefficients. Income elasticity is η_i and the uncompensated price elasticity of η_{ij} goods than the price of j^{th} goods is as follows:

$$\eta_i = \frac{\beta_i}{w_i} - 1 \qquad (66)$$

$$\eta_{ij} = \frac{s_{ij}}{w_i} - \eta_i w_j \qquad (67)$$

CNS model has been indicated in a differential form in equation (64) that should be changed into limited changes to reach estimable equations. Tile method can be used for Rotterdam model that is principally an application of trapeze rule. Two period weighting mean is used for budget proportion.

$$\overline{w}_{jt} = (w_{j,t-1} + w_{j,t})/2 \qquad (68)$$

And logarithmic differential operator D will be as follows:

t=2,000,T $Dy_t = \ln y_t - \ln y_{t-1}$ (69)

After inserting the disturbance sentence of ε_{it}, the expression of limited changes will be as follows:

$$\overline{w}_{jt} D\frac{q_{jt}}{Qt} = \beta_i DQ_t + \sum_{j=1}^{n} s_{ij} DP_j + \varepsilon_{jt} \qquad (70)$$

Where DQt will be computed as $\sum_j \overline{w}_{jt} Dq_{jt}$ that guarantees the additive feature:

The introduced CBS model is a microeconomic equation for individual households. This model is a differential demand system that is used for measuring the impact of Ghiken changes and the total expenditure on budget proportion of various goods. Therefore, this form of model is useful for time series analyses. For the sectional information analyses of demand model, it is better to be a level instead of discriminating or differential. This issue has been studied by Dreil (1982) and Dreil, Nadal and Zilenburgh.

This system includes AIDS income coefficient and S_{ij} Rotterdam price coefficients. This coefficients shares only two basic model in homogeneous additive condition and the difference in the only coefficients. This model can also be made by negative condition; in other words, this condition can also be imposed on the model. According to equations (46) and (47), b_i is appeared instead of C_i. A complete independence and strong and weak separation are not specific states of this specification.

Nous studied another hybrid model named NBR. He replaced $w_i - b_i$ for C_i in AIDS demand system and NBR demand system will be obtained as follows:

$$dw_i + w_i d\ln Q = b_i d\ln Q + \sum_j r_{ij} d\ln p_j \quad (71)$$

This system keeps Rotterdam income coefficients and AIDS price coefficients constant. This model also meets additive rule conditions, homogeneity and symmetry, but it cannot provide negative condition. Moreover, specific preferences structure cannot be inserted by constant choices.

The right side of four partite systems includes similar variables, but the left sides are different. If the left sides of Rotterdam systems (59), CBS system (64), AIDS system (60) and NBR system (71) are shown with y_{Ri} ، y_{Ci} y_{Ai} ، y_{Ni}. Their mutual differences can be shown as follows:

$$y_{ci} - y_{Ri} = w_i(d\ln q_i - d\ln Q) - w_i d\ln q_i = -w_i d\ln Q \quad (1\text{-}72)$$

$$y_{Ai} - y_{ci} = dw_i + w_i(d\ln q_i - d\ln Q) = w_i(d\ln p_i - d\ln p) \quad (2\text{-}72)$$

$$y_{Ni} - y_{Ai} = dw_i + w_i \ln Q - dw_i = w_i d\ln Q \quad (3\text{-}72)$$

In that equations (61) and (21) have been used. All dual differences can be obtained from these three expressions. Traditional demand system of left side variabels, i.e. dlnQ of real income changes, considers $d\ln p_{i\ i=1,000,n}$ as extrovert changes.

Empirical background

Various studies have been done regarding demand systems. One of the most basic models in the field of demand systems is linear expenditure system that was first proposed by Estiven Vajri in 1954. Followed by this model, Hakerter presented indirect logarithmic demand system in 1960. Tile proposed Rotterdam demand system and then mentioned flexible consequential forms of demand system followed by Divert article. Deiton and Malbouser proposed Almost Ideal Demands System in 1980 in that Melina estimated Spain Nutrition Demand during 1964-1985 using this model. Dreil and Keler presented a combinational model of Almost Ideal Demand System and Rotterdam Demand System for the first time from Central Statistics Organization of Netherlands that is known as cbs demand system. Another combinational model was proposed by Nous in 1987 that is known NBR model. these models have been studied as a case study in that the studies conducted by Khosroujenad, (1990), Samimifard, (1993), Adivi (1993), Abdoli (1996), Panahi, (1998) and Muhammadzade, (2003) etc. are of these kinds.

Statistics and Information Used for Model Evaluation

To empirically review Rotterdam demand system, annual data, expenditures consumed by urban households in Sistan and Balouchestan province during 1985-2009 have been used. Given that the main purpose in this study was to empirical study of models and testing homogeneous conditions and symmetry, then five groups have been used instead of 8 groups of goods and services as follows:

- Groups of meals, drinks and drugs (KH)
- Groups of raiment and shoes (PO)
- Groups of settlements and baggage (AS)
- Miscellaneous groups (MO)

It should be noted that groups of miscellaneous goods have been obtained from the total of commodity groups of transportation, health, pastime and miscellaneous. To obtain the price index of this group, Eston index has also been used.

Empirical study of Rotterdam demand system

In this part of article, demand system mentioned above will be reviewed. Firstly, demand will be met and then the theoretical properties of consumer will be tested.

Rotterdam Demand Systems by applying some conditions

In this section, Rotterdam demand system is evaluated by applying some conditions. By applying conditions, it means that homogeneous conditions like $\sum_{j=1}^{5} s_{ij} = 0$ and symmetry $s_{ij} = s_{ji}$ are applied to the model. regarding the fact that five commodity groups are considered to assess the model, Rotterdam model can be written for the first four groups as follows:

$$w_1 d1q_1 = b_1 d1Q + \sum_{j=1}^{5} s_{1j}\, d1p \quad j$$

$$w_2 d1q_2 = b_2 d1Q + \sum_{j=1}^{5} s_{2\ j} d1p \quad j$$

$$w_3 d1q_3 = b_3 d1Q + \sum_{j=1}^{5} s_{3\ j} d1p \quad j$$

$$w_4 d1p_4 = b_4 d1Q + \sum_{j=1}^{5} s_{4\ j} d1p \quad j$$

Numbers 1-4 indicates nourishment, raiment, housing and luggage groups, respectively. For simplicity, d1 is shown with D. Now if these four groups are added up together, we will have:

$$\sum_{i=1}^{4} w_i.Dq_i = \sum_{i=1}^{4} b_i.DQ + \sum_{j=1} s_{ij}\ 1n\, p_j$$

The above equation can be written as follows:

$$DQ - w_5 D q_5 = (1 - b_5) DQ + S_{5\ j}\ DP_j$$

This equation are deduced from this place that because $\sum_{j=1}^{5} b_i = 1$, then we have $b_5 = 1 - \sum_{i=1}^{4} b_i$, since $DQ = \sum_{i=1}^{5} w_i Dq_i$, then we have $\sum_{i=1}^{4} Dq_i = DQ - w_5 Dq_5$. Now if the equation is made simpler, the following equation can be obtained:

$$w_5\, Dq_5 = b_5\, Dq_5 + \sum_{j=1}^{5} + S_5 j$$

This equation means that the total of the first four equations is the same as 5th equation and this equation can be deduced from the first four equations. To assess the above model, the seemingly unrelated regression method (SUR) is used. Results of evaluation are given in Table (1):

Table 1. Evaluation of Rotterdam demand system parameters by applying constraints for five groups of consumption expenditure goods, urban consumers in Sistan and Balouchetsan province during 1985-2009

Determination coefficients	Diwizia quantitative index coefficients	Price indexes coefficients					Intercept	Commodities groups
R^2	b_i	S_5	S_{i4}	S_{13}	S_{12}	S_1		
0/96	0/50 (0/02)	0/07 (0/03)	0/12 (0/04)	0/14 (0/047)	-0/04 (0/028)	-0/14 (0/032)	0/01- (0/007)	Kh (se)
0/96	0/12 (0/007)	0/13 (0/011)	0/051 (0/011)	0/021 (0/013)	-0/02 (0/008)	-0/058 (0/008)	-0/002 (0.002)	Po (se)
0/97	0/18 (0/009)	0/041 (0/032)	-0/075 0/028	-0/19 (0/033)	0/041 (0/019)	0/083 (0/255)	0/003 (0/0046)	Mas (se)
0/98	0/04 (0/011)	0/041 (0/015)	-0/108 (0/016)	-0/004 (0/017)	0/024 (0/011)	0/031 (0/011)	0/001 (0/002)	As (se)

Regarding the parameters related to miscellaneous commodity groups demand, the evaluated coefficients for four commodity groups and homogenous and symmetry are used. The coefficients related to price indexes are deduced from the following equations:

$S_{11}+S_{12}+S_{13}+S_{14}= -S_{15}=-S_{51}$

$S_{21}+S_{22} + S_{23} + S_{24}= - S_{25} = - S_{52}$

$S_{31} + S_{32} + S_{33} + S_{34} = - S_{35} = - S_{53}$

$S_{41} + S_{42} + S_{43} + S_{44} = - S_{45} = -S_{54}$

Generally, the above equations can be written as follows:

$$\sum_{i=1}^{4}\sum_{j=1}^{5} s_{5\ j}$$

According to the above equations, deduction coefficients for miscellaneous goods will be as follows:

Diwizia quantitative index coefficients	Price indexes coefficients					Commodity groups
b_i	S_{i5}	S_{i4}	S_{i3}	S_{i2}	S_{i1}	
0/016	-0/165	0/012	0/031	-0/005	0/082	mo

According to deduced parameters, domestic price elasticity $(\varepsilon_{ij}\ , i = j)$, interceptor price elasticity $(\varepsilon_{ij}\ , i \neq j)$, and income elasticity (η_i) can be computed. Results of computations are given in Table (2):

Table 2. Price elasticity (ε_{ij}) and income elasticity (η_i) of five different commodity groups deduced from Constrained Rotterdam demand system

	ε_{i1}	ε_{i2}	ε_{i3}	ε_{i4}	ε_{i5}	η_i
Nutrition commodity group	-0/39	-0/11	0/39	0/34	0/2	1/4
Raiment and shoes commodity group	-0/52	-0/18	0/19	0/46	0/12	1/07
Housing commodity group	0/29	0/14	-0/63	-0/27	0/14	0/64
Luggage commodity groups	0/46	0/35	-0/04	-1/58	0/6	0/59
Miscellaneous commodity group	0/47	-0/03	0/18	0/07	-0/92	0/89

With respect to Table (2), it can be stated that all goods provided demand rule and have negative domestic price elasticity. In this regard, price elasticity of luggage commodity group is -1.58 which had the highest reaction to price changes among commodity groups. Income elasticity of nutrition is 1.4. This elasticity indicates that the expenditures of commodity group of urban nourishment increases 1.4% by increasing the expenditures of urban households in Sistan and Blaouchestan which this issue indicates that the budget proportion of nourishment group increases by increasing income or urban household expenditure.

If the estimated demand equations are going to be approved theoretically, homogeneity and symmetry tests are conducted for Rotterdam equations system. Wald test is used for testing homogeneity and symmetry in that its results are given in Tables (3) and (4).

Table 3. Test of homogeneity hypothesis of Rotterdam system demand equations with applying conditions

	Test statistics	Critical value	Probability level
Nutrition commodity group	Chi - square	9/95	0/0016
Raiment and shoes commodity group	Chi - square	0/32	0/58
Housing commodity group	Chi - square	14/2	0/0002
Luggage commodity groups	Chi - square	0/89	0/35

According to table (3), it can be indicated that homogeneity features regarding nourishment and housing commodity groups are rejected at the significant level of 5%. Concerning raiment and luggage groups, the homogeneity features cannot be rejected based on current observations at the significant level of 5%.

Table 4. Symmetry hypothesis test in Rotterdam demand system by applying constraints

	Test statistics	Critical value	Probability level
Rotterdam demand system by applying constraints	Chi - square	16/35	0/012
Standard deviation		Critical value	Normalized limitations
0/029		0/0084	C(12)-C(21)
0/053		0/056	C(13)-C(31)
0/039		0/094	C(14)-C(41)
0/023		-0/02	C(23)-C(32)
0/014		0/027	C(24)-C(42)
0/034		-0/072	C(34)-C(43)

Symmetry features about Rotterdam demand system will not be established with applying constraints; that is, this assumption that (sij=sji) about Rotterdam demand system based on the statistics delivered in Aazarbaijan Gharbi province at the significance level of 5% is not confirmed.

Empirical Study of non-constrained Rotterdam demand system

By non-constrained model, it means that symmetry conditions are not inserted in Rotterdam demand system. Therefore, non-homogeneous condition ($\sum_{i=1}^{5} s_{ij} = 0$) is involved in the model. To involve this condition in the model's evaluation, for example, the first equation, it is acted as follows:

$$s_{11} + s_{12} + s_{13} + s_{14} + s_{15} = 0 \Rightarrow s_{15} = -s_{11} - s_{12} - s_{13} - s_{14}$$

Consider the following equation that indicates Rotterdam demand system for nourishment goods group:

$$w_1 Dq_1 = b_1 dQ + s_{11} Dp_1 + s_{12} Dp_2 + s_{13} Dp_3 + s_{14} Dp_4 + s_{15} Dp_{15}$$

In the above equation, index (1) indicates nourishment commodity group and the other ones indicates nourishment, housing, luggage and miscellaneous goods, respectively. When the above condition is inserted, we will have:

$$w_1 Dq_1 = b_1 dQ + s_{11} Dp_1 + s_{12} Dp_2 + s_{13} Dp_3 + s_{14} Dp_4 + (-s_{11} - s_{12} - s_{13} - s_{14}) DP_{15}$$

Now if the above equation is simplified, we will have:

$$w_1 Dq_1 = b_1 dQ + s_{11}\{Dp_1 - Dp_5\} + s_{12}\{Dp_2 - Dp_5\} + s_{13}\{Dp_3 - Dp_5\} + s_{14}\{Dp_4 - Dp_5\}$$

Finally, non-constrained Rotterdam demand system for five commodity groups will be as follows:

$$w_i Dq_i = b_i DQ + \sum s_{ij}\{Dp_j - Dp_5\} + \varepsilon_{it}$$

To assess the above model, seemingly unrelated regression method (SUR) is used. The assessment results are given in Table (5):

Table 5. Eevaluation of non-constrained Rotterdam demand system for five commodity groups of consumption expenditure, urban consumers of Sistan and Blaouchestan during 1985-2009

Determination coefficients	Diwizia quantitative index coefficients	Price indexes coefficients				Intercept	Commodities groups
kh	0/001	S_{i1}	S_{i2}	S_{i3}	S_{i4}		
Po	-0/001	-0/16	-0/04	0/03	-0/14 (0/032)	B_i	R^2
Mas	-0/004	-0/05	-0/018	0/014	-0/058 (0/008)	0/45	0/93
As	-0/001	0/097	0/034	-0/13	0/083 (0/255)	0/12	0/94
*Mo	-	0/31	0.023	0/008	0/031 (0/011)	0/22	0/86

It is deduced from calculations

Given the deduced parameters, domestic price elasticity $(\varepsilon_{ij}, i = j)$, interceptor one $(\varepsilon_{ij}, i \neq j)$ and income elasticity (η_i) can be calculated through $(\varepsilon_{ij} = \frac{s_{ij}}{w_i}, \eta_i = \frac{b_i}{w_i})$. Results of calculations are given in Table (6).

Table 6. Price elasticity (ε_{ij}) and income (η_i) of five different groups of deduced goods from non-constrained Rotterdam demand system

	ε_{i1}	ε_{i2}	ε_{i3}	ε_{i4}	η_i
Nutrition commodity group	-0/45	-0/11	0/08	0/25	1/4
Raiment and shoes commodity group	-0/45	-0/16	0/12	0/38	1/07
Housing commodity group	0/34	0/12	-0/42	-0/14	0/64
Luggage commodity groups	4/6	0/34	0/12	-0/58	0/59
Miscellaneous commodity group	-1/09	0/01	0/38	-0/29	0/89

Given table (6), it can be indicated that all commodity groups meet demand rule and have negative domestic price elasticity. In this respect, the price elasticity of luggage commodity group equals -0.58 which shows the highest reaction to price changes among commodity groups. The income elasticity of nourishment group equals 1.2. This elasticity indicates that the commodity group expenditure of urban nourishment increases as 1.2% by

increasing urban household expenditures in Sistan and Balouchestan province which this issue indicates that the budget proportion of nourishment group increases by increasing income or urban household expenditures.
In order for the estimated demand equations are approved theoretically, homogeneity and symmetry test are conducted for Rotterdam demand equations system. To test the homogeneity and symmetry conditions, Wald test is used which its results are given in Tables (7) and (8).

Table 7. Test of homogeneity hypothesis of system demand equations of Rotterdam demand by applying constraints

	Test statistics	Critical value	Probability level
Nutrition commodity group	Chi - square	2/1	0/145
Raiment and shoes commodity group	Chi - square	2/1	0/145
Housing commodity group	Chi - square	1/069	0/3
Luggage commodity groups	Chi - square	6/99	0/008

According to table (7), it can be indicated that the homogeneity feature about nourishment commodity groups, housing and raiment at the significant level of 5% cannot be rejected with respect to Chi-2 statistics and the homogeneity assumption is also approved. Homogeneity features about luggage commodity group are rejected at the significant level of 5%.

Table 8. Symmetry hypothesis test in Rotterdam demand systems by applying constraints

	Test statistics	Critical value	Probability level
Rotterdam demand system by applying constraints	Chi - square	54.7	50.4
Standard deviation		Critical value	Normalized limitations
0/0374		0/0135	C(12)-C(21)
0/0535		-0/0616	C(13)-C(31)
0/054		0/064	C(14)-C(41)
0/0319		-0/0201	C(23)-C(32)
0/0497		-0/0573	C(34)-C(43)
0/0374		0/0135	C(12)-C(21)

Symmetry features in non-constrained Rotterdam demand system is maintained, i.e. this assumption that (sji= sij) is approved in non-constrained Rotterdam demand system based on Sistan and Balouchestan information statistics at the significant level of 5%.

CONCULSION

This aim of this study was to examine deduction methods of demand systems for the consumer behavior. With respect to the purpose of article, Rotterdam demand system was evaluated in constrained and non-constrained methods using annual consumption expenditure data of Sistan and Balouchrstan urban households during 1985-2009. Results obtained from model evaluation and hypotheses test related to consistency with theoretical features of consumers' behavior showed that homogeneity condition is established in both constrained and non-constrained states in Rotterdam demand system and symmetry condition is not true in constrained state while it is approved in non-constrained state.

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